Dynamic Textures Synthesis for Probing Vision in Psychophysics and Electrophysiology

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PhD Defense
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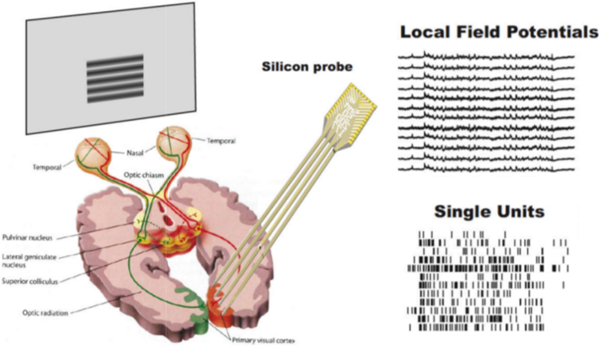
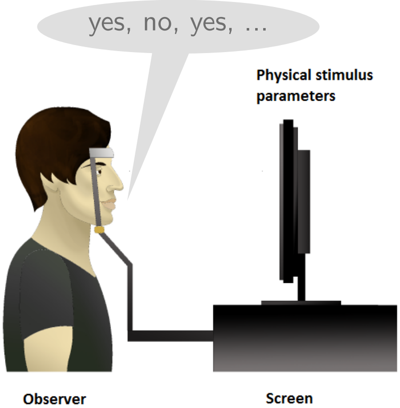
Context: Electrophysiology and Psychophysics

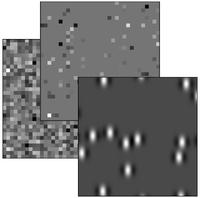




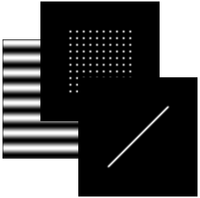
stimulus $\rightarrow$ $\boxed{??}$ $\rightarrow$ responses

Goals

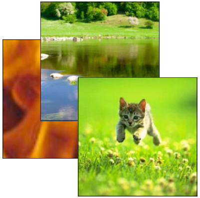
Noise



Artificial



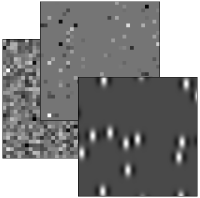
Natural



- Develop a parametric model of dynamic visual stimulation;

Goals

Noise



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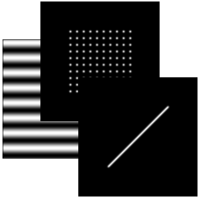

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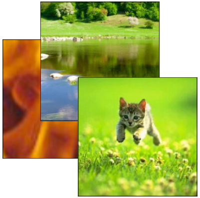

- Develop a parametric model of dynamic visual stimulation;
- Run experiments in psychophysics and electrophysiology;

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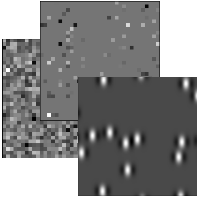

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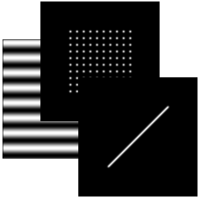

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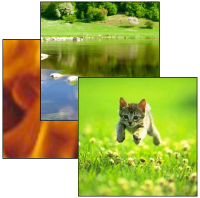

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- Build mathematical models;

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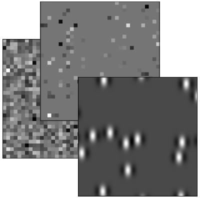

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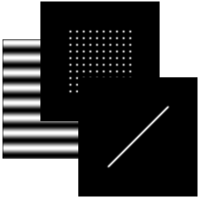

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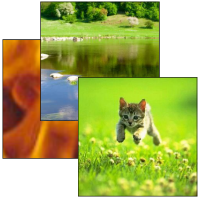

- Develop a parametric model of dynamic visual stimulation;
- Run experiments in psychophysics and electrophysiology;
- Build mathematical models;
- Analyze data using Machine Learning techniques.

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Definition (2D Marked Poisson Process)

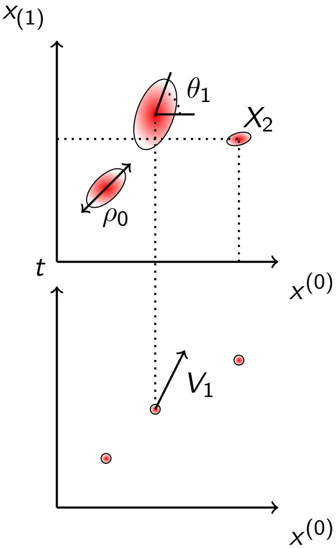
$$I_{\lambda,g}(x,t) = \frac{1}{\sqrt{\lambda}} \sum_{p\in\mathbb{N}} g(\rho_p R_{\theta_p}(x - X_p - V_p t))$$

- $(X_p)_{p\in\mathbb{N}}$ is a 2D Poisson process,
- $(\rho_p, \theta_p, V_p)_{p\in\mathbb{N}}$ are i.i.d. random variables with density $(\mathbb{P}_Z, \mathbb{P}_\theta, \mathbb{P}_V)$.
- $I_{\lambda,g}$ is stationary with bounded second order moments.

Its autocovariance function is $\forall (x,t) \in \mathbb{R}^3$,

$$\gamma(x,t) = \int c_g(\rho R_\theta(x - \nu t))\mathbb{P}_V(\nu)\mathbb{P}_Z(\rho)\mathbb{P}_\Theta(\theta)\mathrm{d}\nu\mathrm{d}\rho\mathrm{d}\theta \quad (1)$$

where $c_g = g \star \bar{g}$.



First MC Formulation: Asymptotic of a Spot Noise

Proposition (Galerne *et al.* [2])

When $\lambda \to +\infty$, $I_{\lambda,g}$ converges toward a stationary Gaussian field I_g of zero mean and autocovariance function γ.

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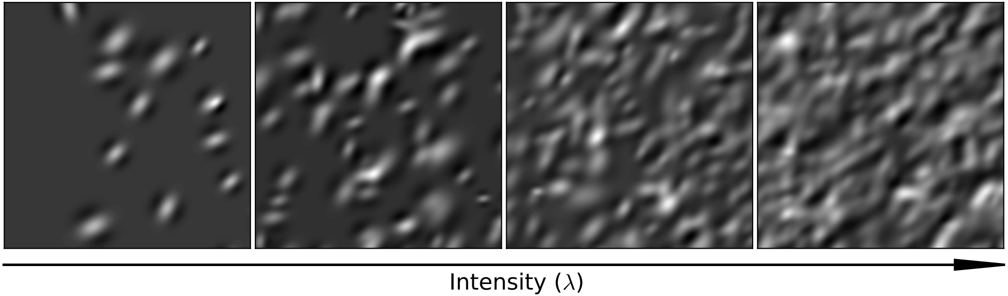
Example:



Second MC Formulation: Power Spectrum

Consider the following texton

$$g_\sigma(x) = \frac{1}{2\pi} \cos\left(\langle x,\, \xi_0 \rangle\right) e^{-\frac{\sigma^2}{2}\|x\|^2}. \tag{2}$$

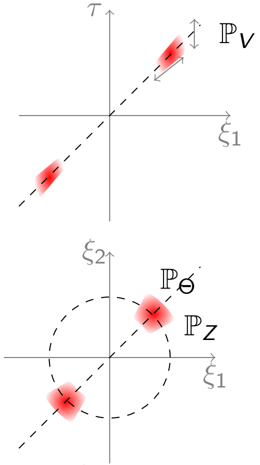
Proposition (Definition of Leon *et al.* [4])

When $\sigma \to 0$, the Gaussian field $I_\sigma(x,t)$ defined in Proposition 1 converges toward a stationary Gaussian field of covariance having the power-spectrum $\forall\,(\xi,\tau) \in \mathbb{R}^2 \times \mathbb{R}$,

$$\hat{\gamma}(\xi,\tau) = \frac{\mathbb{P}_Z\left(\|\xi\|\right)}{\|\xi\|^2} \mathbb{P}_\Theta\left(\angle\xi\right) \mathcal{L}(\mathbb{P}_{\|V-v_0\|}) \left(-\frac{\tau + \langle v_0,\, \xi\rangle}{\|\xi\|}\right), \tag{3}$$

where the linear transform $\mathcal{L}$ is such that

$$\forall\, u \in \mathbb{R}, \quad \mathcal{L}(f)(u) \overset{\text{def.}}{=} \int_{-\pi}^{\pi} f(-u/\cos(\varphi))\mathrm{d}\varphi. \tag{4}$$



Power Spectrum $\hat{\gamma}$.

Third MC Formulation: sPDE

- No average translation $v_0 = 0$;
- Critical regime *ie* $\hat{\alpha}(\xi) = \frac{2}{\hat{\nu}(\xi)}$ and $\hat{\beta}(\xi) = \frac{1}{\hat{\nu}(\xi)^2}$.

$$\frac{\partial^2 I}{\partial t^2} + \alpha \star_s \frac{\partial I}{\partial t} + \beta \star_s I = \frac{\partial W}{\partial t} \quad \underset{\mathcal{F}_s^{-1}}{\overset{\mathcal{F}_s}{\rightleftarrows}} \quad \frac{\partial^2 \hat{I}}{\partial t^2} + \hat{\alpha}\frac{\partial \hat{I}}{\partial t} + \hat{\beta}\hat{I} = \hat{\sigma}_W^2 \frac{\partial \hat{W}}{\partial t} \tag{5}$$

Proposition (V. *et al.* [10])

When considering

$$\forall\, r > 0, \quad \mathbb{P}_{\|V - v_0\|}(r) = \mathcal{L}^{-1}(h)(r/\sigma_V) \quad \textit{where} \quad h(u) = (1 + u^2)^{-2}$$

where $\mathcal{L}$ is defined in (3), equation (5) admits a solution I which is a stationary Gaussian field with power spectrum (3) when setting

$$\hat{\sigma}_W^2(\xi) = \frac{4}{\hat{\nu}(\xi)^3 \|\xi\|^2} \mathbb{P}_Z(\|\xi\|)\mathbb{P}_\Theta(\angle \xi), \quad \textit{and} \quad \hat{\nu}(\xi) = \frac{1}{\sigma_V \|\xi\|}.$$

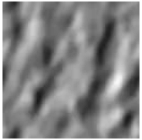
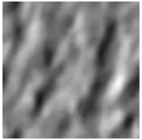
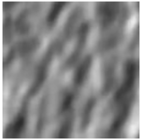
AR(2): Fast Algorithm

Numerically, we estimate Equation (5) over the Fourier domain,

$$\hat{I}^{(\ell+1)}(\xi) = \hat{\mathcal{U}}_{v_0}(\xi)\hat{I}^{(\ell)}(\xi) + \hat{\mathcal{V}}_{v_0}(\xi)\hat{I}^{(\ell-1)}(\xi) + \Delta\hat{\sigma}_W(\xi)(\hat{w}^{(\ell)}(\xi) - \hat{w}^{(\ell-1)}(\xi)),$$

$$\text{where} \quad \begin{cases} \hat{\mathcal{U}}_{v_0}(\xi) \stackrel{\text{def.}}{=} (2 - \Delta\hat{\alpha}(\xi) - \Delta^2\hat{\beta}(\xi))e^{-\mathrm{i}\Delta v_0 \xi}, \\ \hat{\mathcal{V}}_{v_0}(\xi) \stackrel{\text{def.}}{=} (-1 + \Delta\hat{\alpha}(\xi))e^{-2\mathrm{i}\Delta v_0 \xi}, \end{cases}$$

and where $w^{(\ell)} - w^{(\ell-1)}$ is a 2-D white noise with distribution $\mathcal{N}(0, \Delta)$.

$I^{(\ell+1)} = \mathcal{U}_{v_0} \star$  $+\mathcal{V}_{v_0} \star$  $+\Delta$ 

Distributions

We use biologically inspired distributions

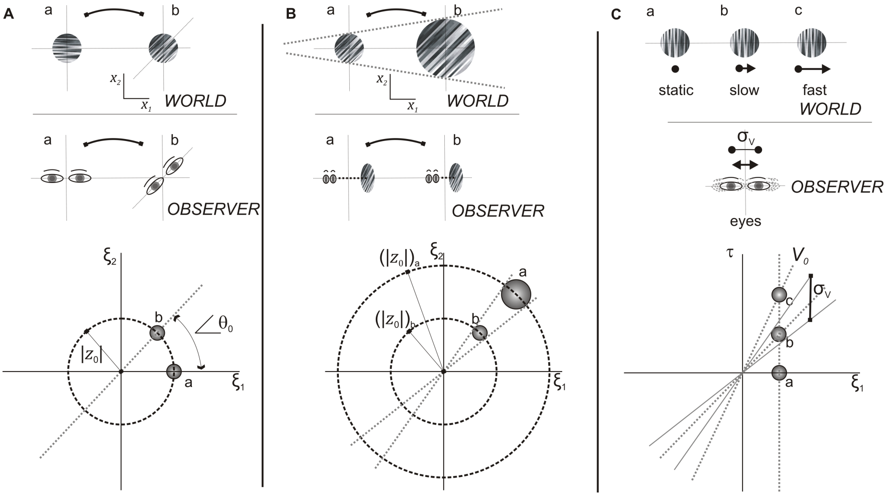
$$\mathbb{P}_Z(z) \propto \frac{z_0}{z} \exp\left(-\frac{\ln\left(\frac{z}{z_0}\right)^2}{2\ln\left(1+\sigma_Z^2\right)}\right), \quad \mathbb{P}_\Theta(\theta) \propto \exp\left(\frac{\cos(2(\theta-\theta_0))}{4\sigma_\Theta^2}\right).$$

Noting $v = v_0 + \delta v$ with $\delta v = r(\cos(\varphi), \sin(\varphi))$

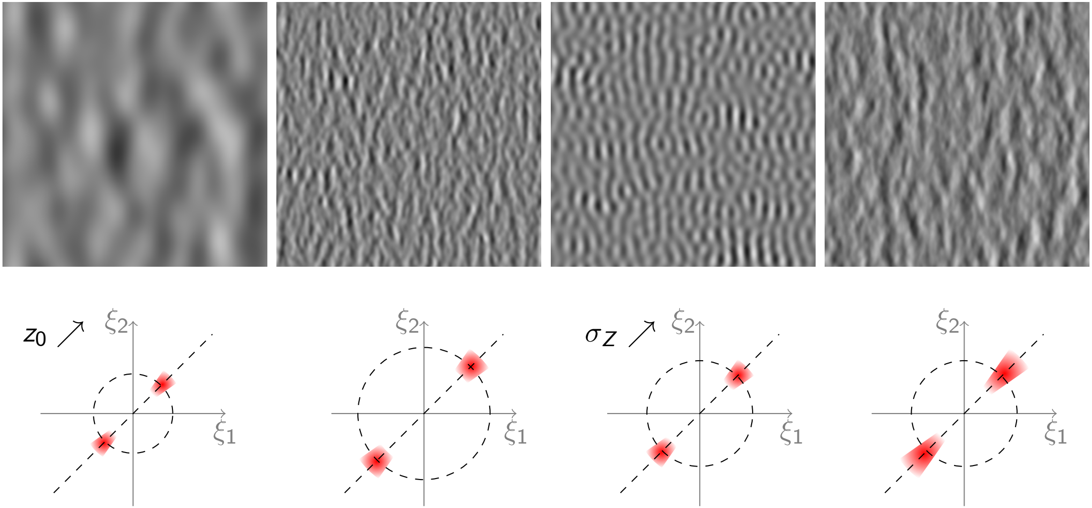
$$\mathbb{P}_{\|V-v_0\|}(r) \propto \frac{r}{r_0} \exp\left(-\frac{\ln\left(\frac{r}{r_0}\right)^2}{2\ln\left(1+\sigma_V^2\right)}\right),$$

	Speed am.	Freq. orient.	Freq. am.
(μ, σ)	(r_0, σ_V)	$(\theta_0, \sigma_\Theta)$	(ρ_0, σ_Z)

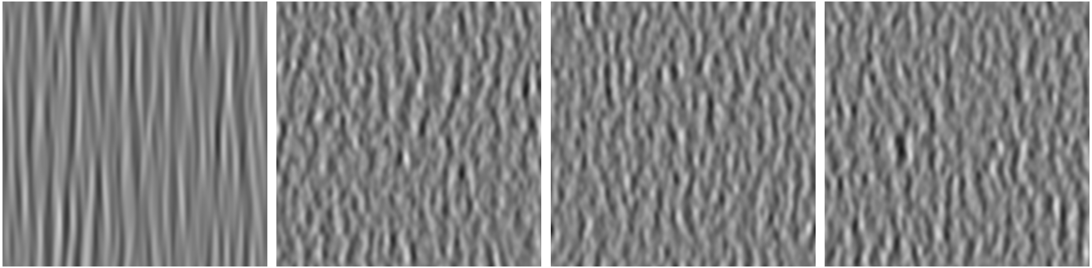
Parameters

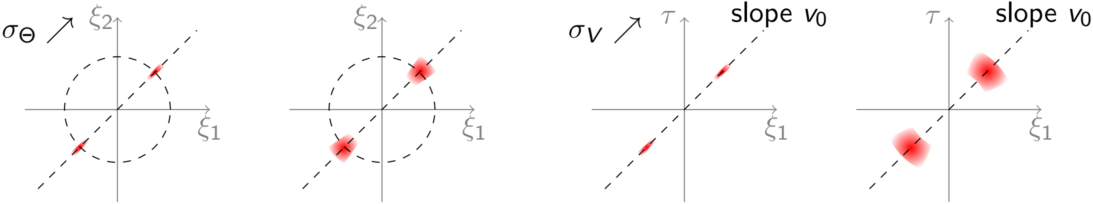


A
a
b
x_2
x_1
WORLD
a
b
OBSERVER
ξ_2
b
θ_0
$|z_0|$
a
ξ_1
B
a
b
x_2
x_1
WORLD
a
b
OBSERVER
$(|z_0|)_a$
ξ_2
a
$(|z_0|)_b$
b
ξ_1
C
a
b
c
static
slow
fast
WORLD
σ_V
OBSERVER
eyes
τ
V_0
c
σ_V
b
a
ξ_1


$z_0 \nearrow$
ξ_2
ξ_1
ξ_2
ξ_1
$\sigma_Z \nearrow$
ξ_2
ξ_1
ξ_2
ξ_1

Examples: Orientation and Speed Distributions





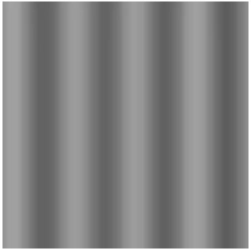
To Go Further

$$\frac{\partial^2 I}{\partial t^2} + \alpha \star_s \frac{\partial I}{\partial t} + \beta \star_s I = \frac{\partial W}{\partial t} \quad \text{where} \quad \frac{\partial W}{\partial t} \sim \mathcal{N}(0, \sigma_W)$$

- Texture Synthesis (Xia *et al.* [12]);

Find estimates $\tilde{\alpha}, \tilde{\beta}, \tilde{\sigma}_W$ from a texture example.

- Trajectories in the space of parameters.



Time dependence: $\alpha(x, t), \beta(x, t), \sigma_W(x, t)$.

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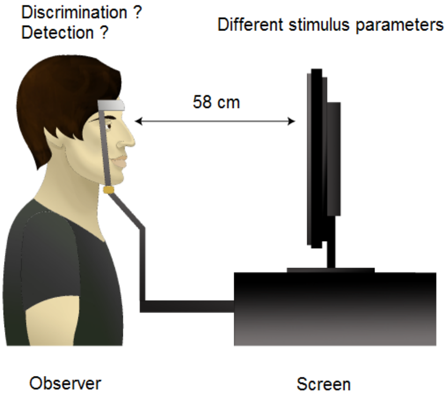
What Is Psychophysics ?

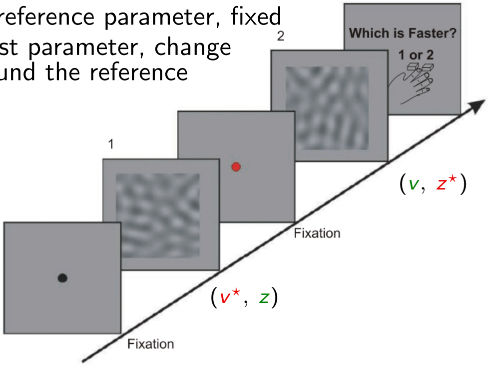


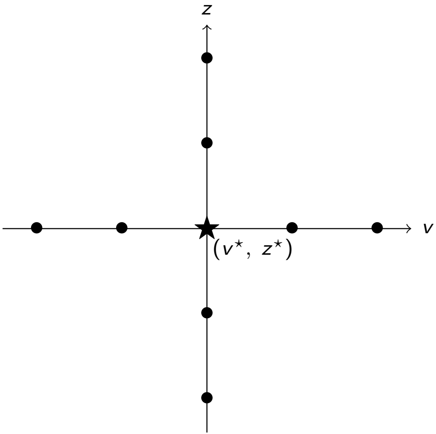
- Make connections between subjective responses and physical parameters;
- Detection, discrimination, response time/delay.

What is the effect of spatial frequency z over perception of speed v ? (Brooks *et al.* [1])

$v^\star, z^\star$: reference parameter, fixed

v, z : test parameter, change around the reference





Experimental protocol

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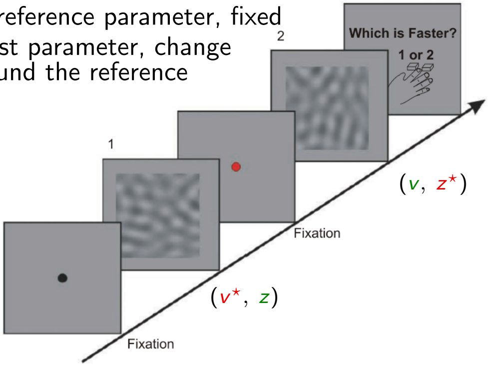
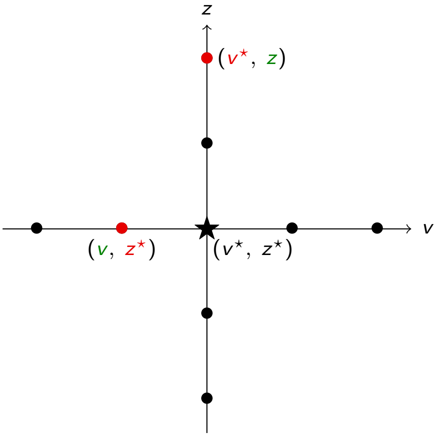
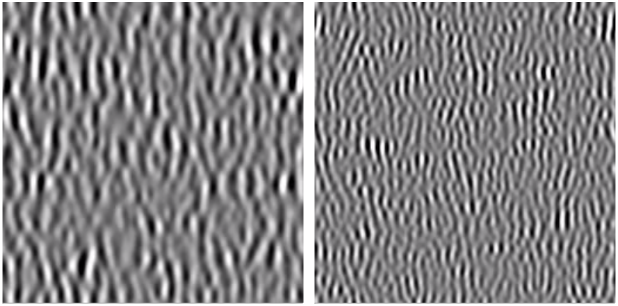




One trial



Psychometric Function (Wichmann *et al.* [11])

Definition (Psychometric Samples)

$$\hat{\varphi}_{z^\star,z}(v, v^\star) \sim \mathcal{B}(n, \varphi_{z^\star,z}(v, v^\star))$$

where $\mathcal{B}(n, \varphi_{z^\star,z}(v, v^\star))$ *is the binomial distribution with* $n \in \mathbb{N}^\star$ *trials and probability* $\varphi_{z^\star,z}(v, v^\star) \in [0, 1]$.

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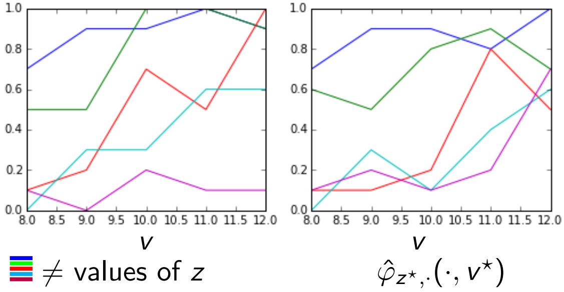

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where f *is a sigmoid-like function and*

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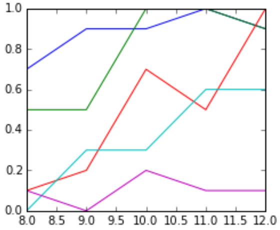
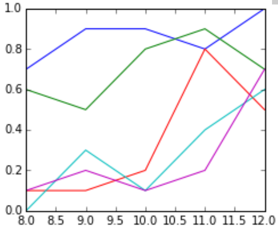


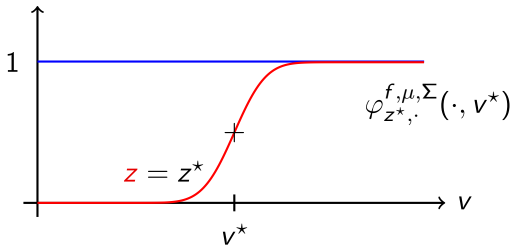
v

$\neq$ values of z



v

$\hat{\varphi}_{z^\star,\cdot}(\cdot, v^\star)$



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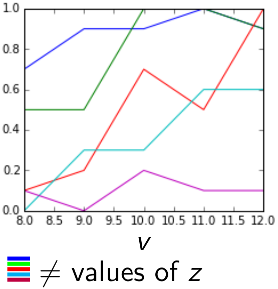

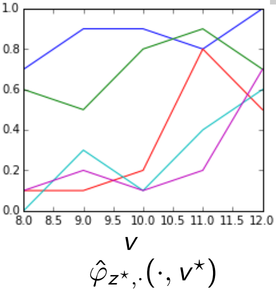

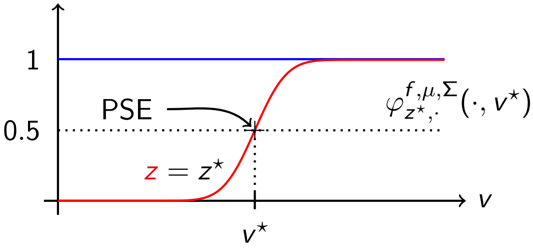

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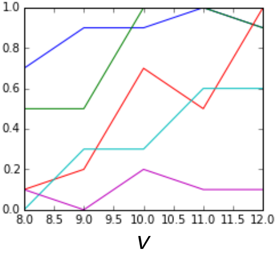

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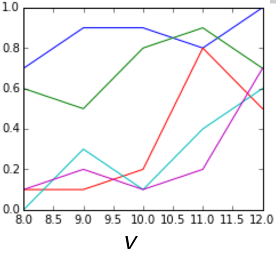
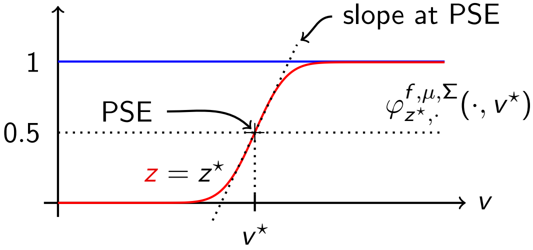


$\hat{\varphi}_{z^\star,\cdot}(\cdot, v^\star)$



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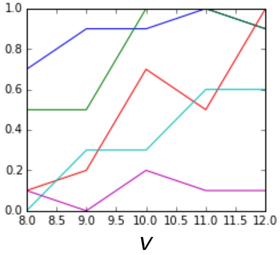
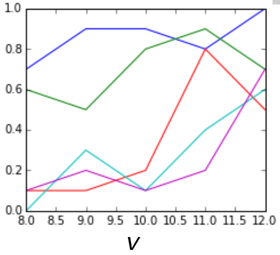
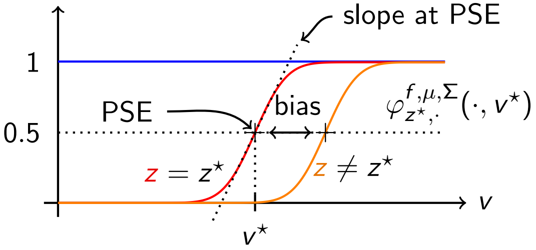


$\neq$ values of z



$\hat{\varphi}_{z^\star,\cdot}(\cdot, v^\star)$



Definition (Psychometric Samples)

$$\hat{\varphi}_{z^\star,z}(v, v^\star) \sim \mathcal{B}(n, \varphi_{z^\star,z}(v, v^\star))$$

where $\mathcal{B}(n, \varphi_{z^\star,z}(v, v^\star))$ *is the binomial distribution with* $n \in \mathbb{N}^\star$ *trials and probability* $\varphi_{z^\star,z}(v, v^\star) \in [0, 1]$.

Definition (Psychometric Function)

$$\varphi_{z^\star,z}(v, v^\star) = \varphi^{f,\mu,\Sigma}_{z^\star,z}(v, v^\star)$$

where f *is a sigmoid-like function and*

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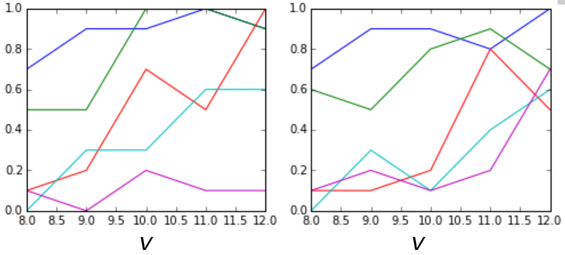

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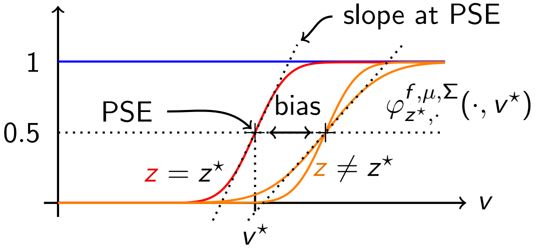
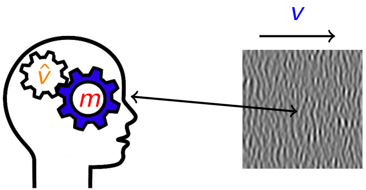


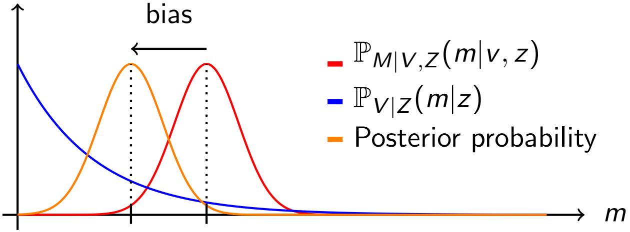
Bayesian Observer (Pouget *et al.* [6])

- The Bayesian observer measures speed v from an internal brain measurement m;
- From these measurement m the Bayesian observer makes an estimation $\hat{v}_z(m)$ of speed using a MAP estimator.

$$\hat{v}_z(m) = \underset{v}{\operatorname{argmax}} \underbrace{\log(\mathbb{P}_{M|V,Z}(m|v,z))}_{\text{likelihood}} + \underbrace{\log(\mathbb{P}_{V|Z}(v|z))}_{\text{prior}}$$



Bayesian observer



Inverse Bayesian Inference

Bayesian Inference:

$$\hat{v}_z(m) = \underset{v}{\operatorname{argmax}} \underbrace{\log(\mathbb{P}_{M|V,Z}(m|v,z))}_{\text{likelihood}} + \underbrace{\log(\mathbb{P}_{V|Z}(v|z))}_{\text{prior}}$$

- Goal (inverse): find likelihood $\mathbb{P}_{M|V,Z}$ and prior $\mathbb{P}_{V|Z}$ knowing estimates $\hat{v}_z(m)$ (Stocker *et al.* [8], Sotiropoulos *et al.* [7]).

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- Drawback: only comparisons $\hat{v}_z(m) > \hat{v}_z^\star(m)$ are accessible from experiment.

Bayesian Inference:

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- Drawback: only comparisons $\hat{v}_z(m) > \hat{v}_z^\star(m)$ are accessible from experiment.

To simplify we assume parametric likelihood and prior:

$$\mathbb{P}_{M|V,Z}(m|v,z) \propto e^{-\frac{|m-v|^2}{2\sigma_z^2}} \quad \text{and} \quad \mathbb{P}_{V|Z}(m|z) \propto e^{a_z m} 1_{[0,v_{\max}]}(m).$$

The Bayesian Psychometric Function

Definition (Psychometric Function)

In the Bayesian model, the psychometric function is defined as

$$\varphi_{z^\star,z}(v^\star, v) \stackrel{\text{def.}}{=} \mathbb{E}(\{\hat{v^\star}_z(m^\star) > \hat{v}_{z^\star}(m)\}|v^\star, v).$$

Proposition

Under the hypothesis of a Gaussian likelihood and a Laplacian prior one has

$$\varphi_{z^\star,z}(v^\star, v) = \varphi^{f,\mu,\Sigma}_{z^\star,z}(v^\star, v)$$

where

$$f(\cdot) = \frac{1}{2}(1 + \mathrm{erf}(\cdot)), \quad \mu_{z,z^\star} = a_{z^\star}\sigma^2_{z^\star} - a_z\sigma^2_z \quad \textit{and} \quad \Sigma_{z,z^\star} = \sqrt{\sigma^2_{z^\star} + \sigma^2_z}$$

Inverse Bayesian Inference Algorithm

Algorithm

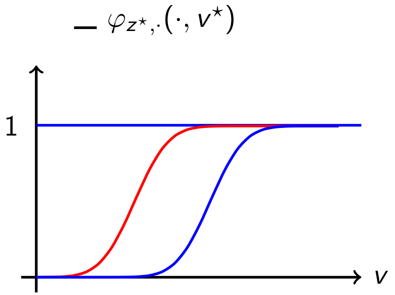
- *Minimize the likelihoods for each pair* $(z, z^\star)$*:*

$$\min_{\mu,\Sigma} \sum_{v} KL(\hat{\varphi}_{z^\star,z}(v^\star, v) | \varphi^{\mu,\Sigma}_{z^\star,z}(v^\star, v))$$

$$\textit{where} \quad KL(\hat{p}|p) = \hat{p} \log\left(\frac{\hat{p}}{p}\right) + (1-\hat{p}) \log\left(\frac{1-\hat{p}}{1-p}\right);$$

- *Solve* $(\mu, \Sigma^2) = M(a, \sigma^2)$*;*
- *Minimize the global likelihood:*

$$\min_{a,\sigma} \sum_{z,z^\star} \sum_{v} KL(\hat{\varphi}_{z^\star,z}(v^\star, v) | \varphi^{a,\sigma}_{z^\star,z}(v^\star, v)).$$



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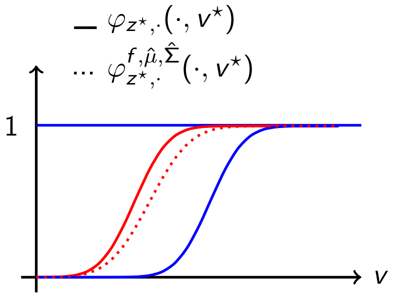

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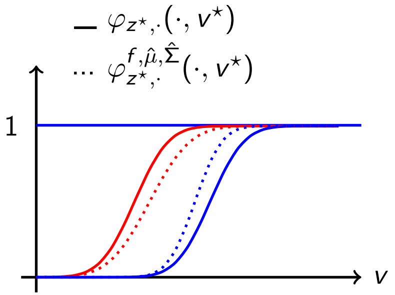

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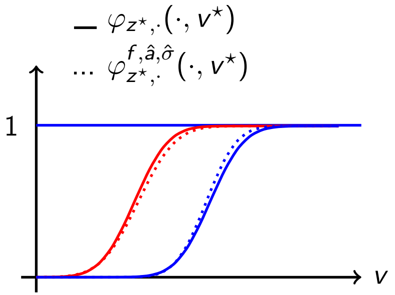
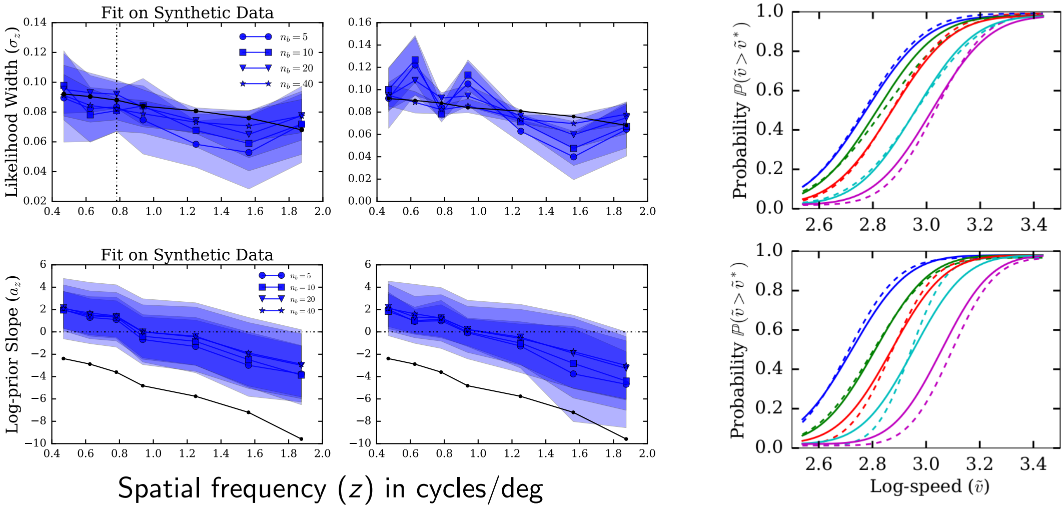
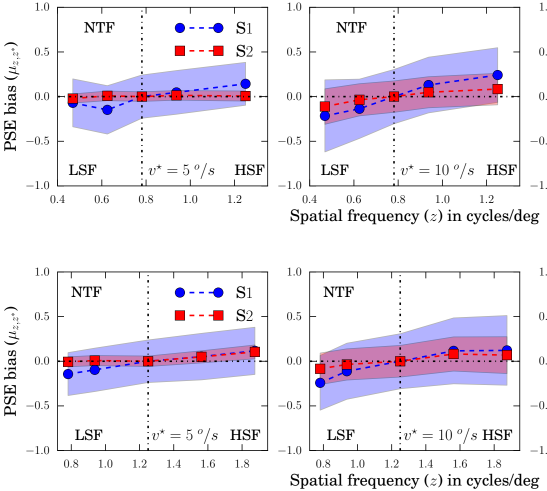


Synthetic Data

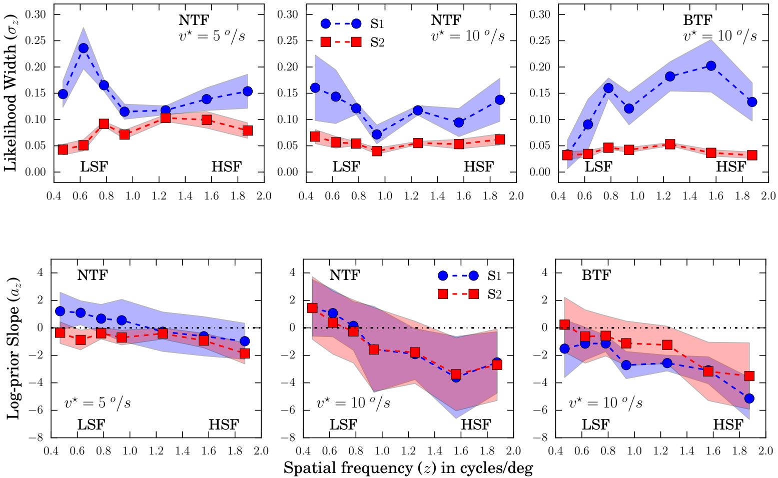


Real Data



- Spatial freq. has a positive effect on perceived speed;

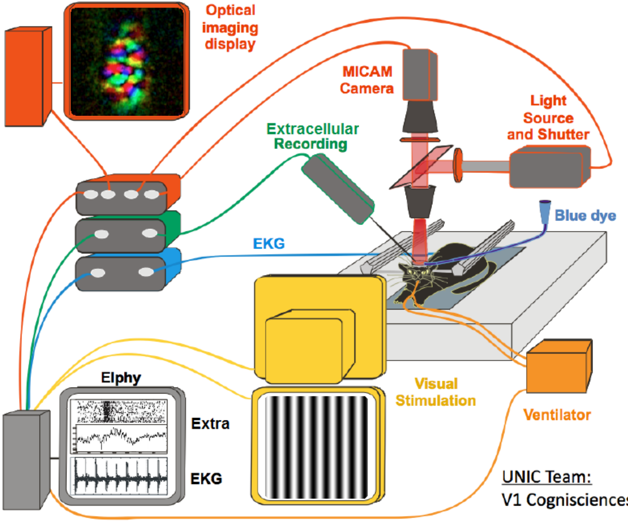
Real Data



- ▶ Spatial freq. has a positive effect on perceived speed;
- ▶ A change in log-prior slope is necessary to explain this effect.

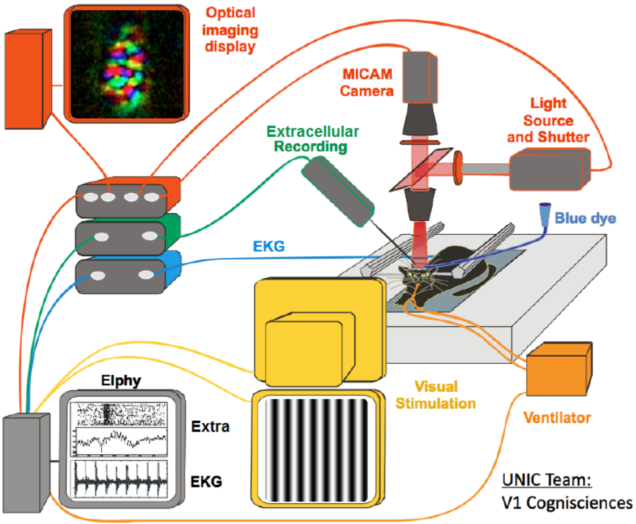
Table of contents

Electrophysiology and Optical Imaging



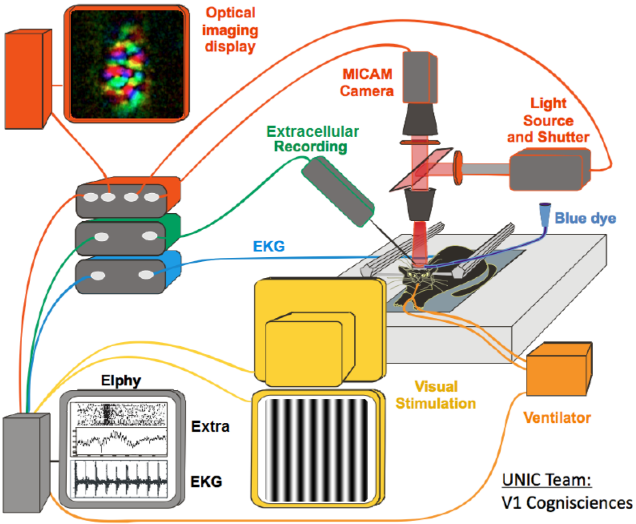
UNIC Team:
V1 Cognisciences

Electrophysiology and Optical Imaging



- Stimulate with a parameter p,

Electrophysiology and Optical Imaging



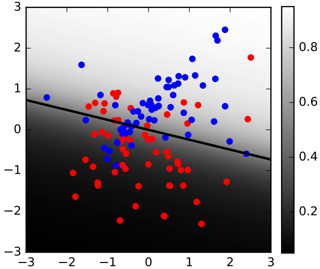
- Stimulate with a parameter p,
- Record a signal s.

Supervised classification: $\forall i \in I, \quad (x_i, y_i) \in \mathcal{X} \times \mathcal{Y}$ where y_i is the class of the feature x_i.

Goal: find a function $f : x \in \mathcal{X} \longmapsto f(x) = y \in \mathcal{Y}$.

Existing work in fMRI: Thirion team [9] and Gallant team [5].

(x, y)



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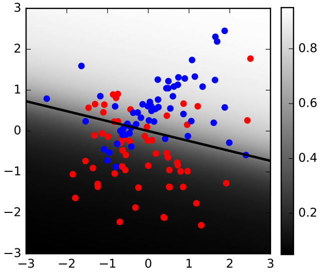

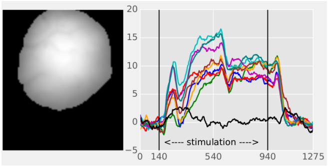

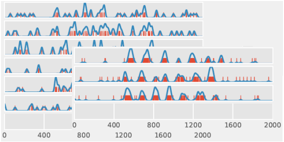

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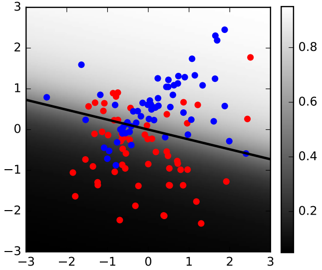
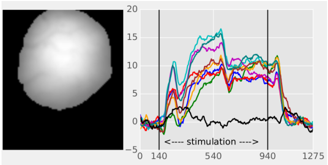
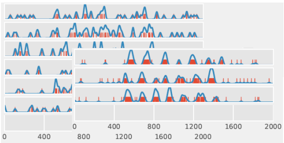






A typical dataset is

$S = (s_{q,t,c,r})_{(q,t,c,r) \in \mathcal{Q} \times \mathcal{T} \times \mathcal{C} \times \mathcal{R}}$

with

$\mathcal{Q}$: pixels or neurons

$\mathcal{T}$: time samples

$\mathcal{C}$: experimental conditions

$\mathcal{R}$: repetitions

$\mathcal{Y} = \mathcal{C}$

$I = \mathcal{T} \times \mathcal{C} \times \mathcal{R}$

or $I = \mathcal{C} \times \mathcal{R}$

Classifiers (Logistic Classification)

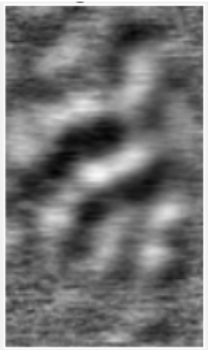
A vector x belongs to class $y \in \mathcal{Y}$ with the following probability:

$$\mathbb{P}_{Y|X,\theta}(y|x) = \frac{e^{\langle x, \omega_y \rangle}}{\sum_{y' \in \mathcal{Y}} e^{\langle x, \omega_{y'} \rangle}}.$$

The estimated weight vectors $(\hat{\omega}_1, \cdots, \hat{\omega}_c)$ are obtained by minimizing

$$\ell(\omega_1, \cdots, \omega_c) = -\sum_{i \in I} \langle x_i, \omega_{y_i} \rangle + \log \left(\sum_{y' \in \mathcal{Y}} e^{\langle x_i, \omega_{y'} \rangle} \right)$$

A typical weight vector obtained on VSD recordings



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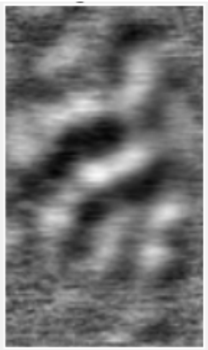
Other classifiers:

- Linear and Quadratic Discriminant Analysis (LDA/QDA),
- Gaussian Naive Bayes (GNB),
- Nearest Centroid (NC).

A typical weight vector obtained on VSD recordings



Evaluation of Classification Performances: Cross-Validation

Definition (n_{folds} Cross-Validation)

Dataset splitting

$$I = \cup_{i=1}^{n_{folds}} I_{test}^{(i)} \quad \textit{with} \quad \forall i \neq j, \quad |I_{test}^{(i)}| = |I_{test}^{(j)}| \quad \textit{and} \quad I_{test}^{(i)} \cap I_{test}^{(j)} = \emptyset$$

Learn on $I_{train}^{(i)} = I \backslash I_{test}^{(i)}$. *Make predictions on* $I_{test}^{(i)}$ *(* $\forall i, \quad \hat{y}_i = f(x_i)$ *).*

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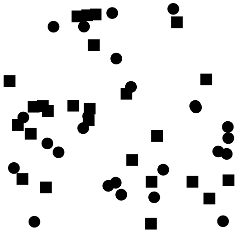
- ■ Square class
- ● Circle class



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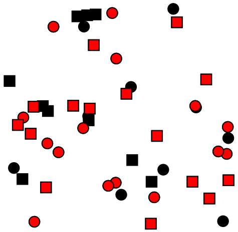
$I_{\text{train}}^{(i)}$



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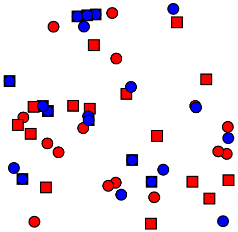
$I_{\text{train}}^{(i)}$ $I_{\text{test}}^{(i)}$



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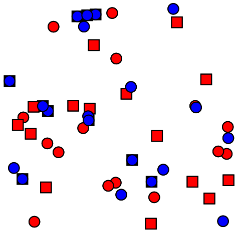
$I_{\text{train}}^{(i)}$ $I_{\text{test}}^{(i)}$

repeat for
$i \in \{1, .., n_{\text{folds}}\}$



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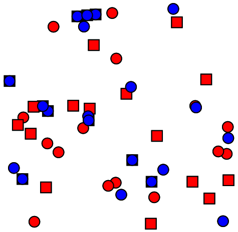
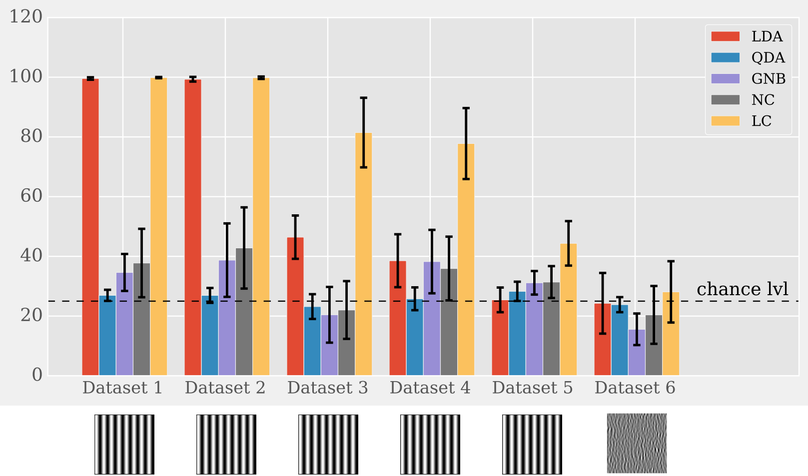


Definition (Score and Av. Score)

$$\iota_{I_{test}} \overset{\text{def.}}{=} \frac{1}{|I_{test}|} \sum_{i \in I_{test}} \delta_{y_i}^{f(x_i)},$$

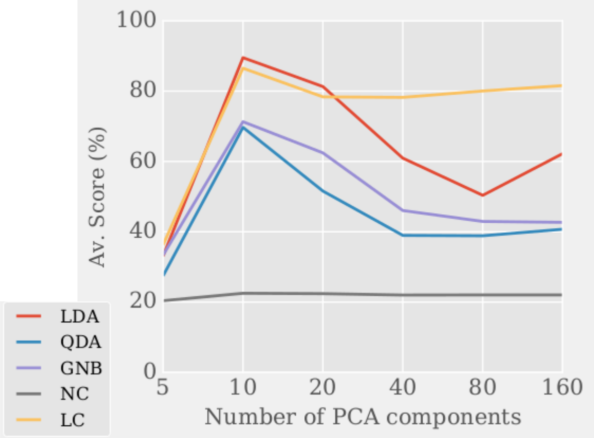
$$\mu_{\iota} \overset{\text{def.}}{=} \frac{1}{n_{folds}} \sum_{i=1}^{n_{folds}} \iota_{I_{test}^{(i)}}$$

VSDi / Comparison of the Different Algorithms



- on raw vectors QDA, GNB and NC fail

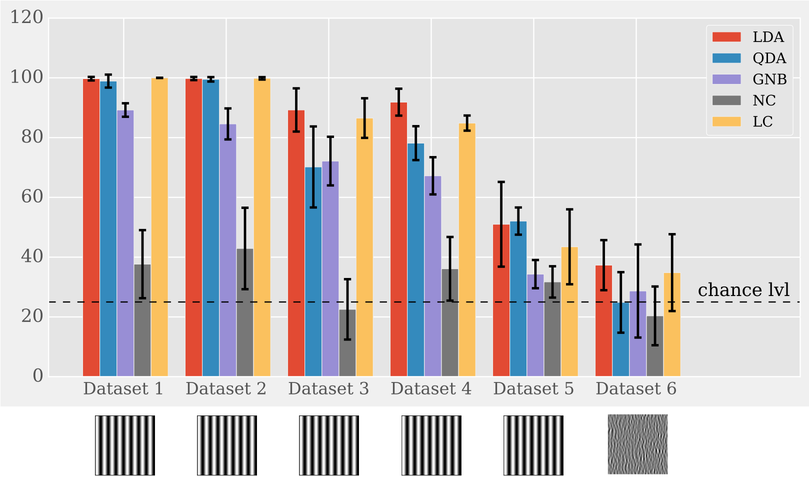
The Principal Component Analysis (PCA) allows for dimension reduction.



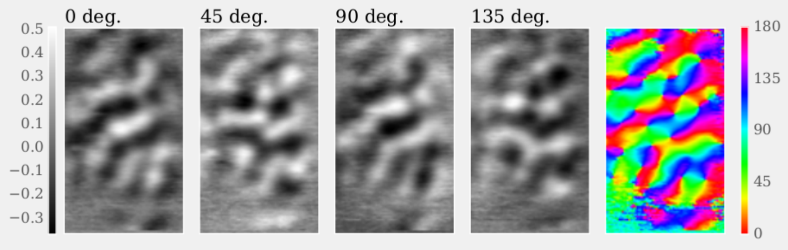
- there often exists a number of PCA components that maximizes the scores
- this number is often between 5 and 160

VSDi / Dimension Reduction and Comparison of Algorithms

The Principal Component Analysis (PCA) allows for dimension reduction.



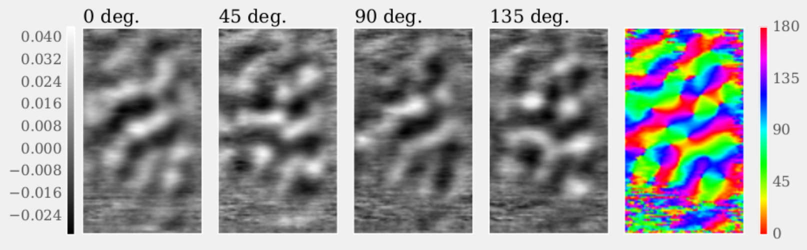
- with dimension reduction
  QDA, GNB show better results
  NC still fail



- PCA + Nearest Centroid

  $m^{(y)}$: activation maps

  o: orientation map



- PCA + Logistic Classif.

  $\forall q \in \mathcal{Q}$,

  $$o_q = \frac{1}{2} \operatorname{Arg}\left(\sum_y m_q^{(y)} \frac{e^{2i\pi\theta_y}}{|\mathcal{Y}|}\right)$$

How to identify highly predictive areas ?

- 2D Gaussian Sliding window

$$\forall q' \in \mathcal{Q}, \quad g_q(q') = \exp\left(-\frac{\|q'-q\|^2}{2\sigma_g^2}\right) \quad \text{and} \quad \mathbb{P}_{Y|X,\theta,q}(y|x) = \frac{e^{\langle x,\, g_q \omega_y \rangle}}{\sum_{y' \in \mathcal{Y}} e^{\langle x,\, g_q \omega_{y'} \rangle}}.$$

VSDi / Spatially Localized Predictions

How to identify highly predictive areas ?

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g_q $\times$ ω_y $=$ $g_q \omega_y$

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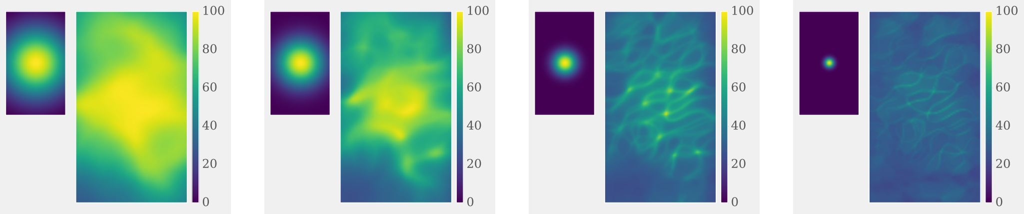
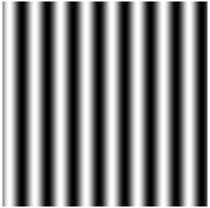
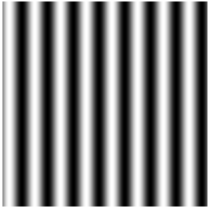


$\sigma_g = 15$ $\sigma_g = 10$ $\sigma_g = 5$ $\sigma_g = 2$

VSDi / Dynamic of Prediction Scores

Protocols:





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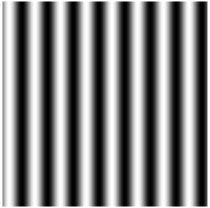

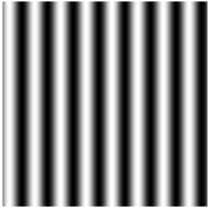

What is the effect of this sharp rotation on the VSD signal ?

- New indexes set

Before: $I = \mathcal{T} \times \mathcal{C} \times \mathcal{R}$

Now: $I_t = \{t\} \times \mathcal{C} \times \mathcal{R}$

Definition (Time Av. Score)

$$\tilde{\mu}_{\iota,t} \stackrel{\text{def.}}{=} \frac{1}{n_{folds}} \sum_{i=1}^{n_{folds}} \iota_{I^{(i)}_{t,test}}$$

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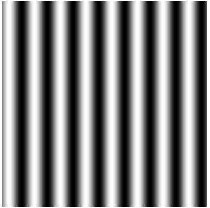

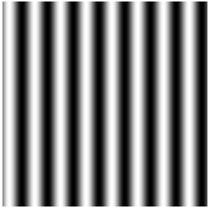

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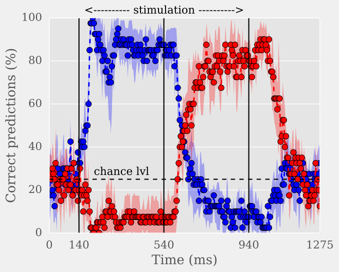
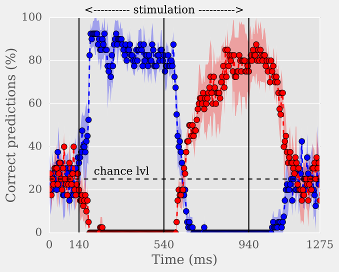
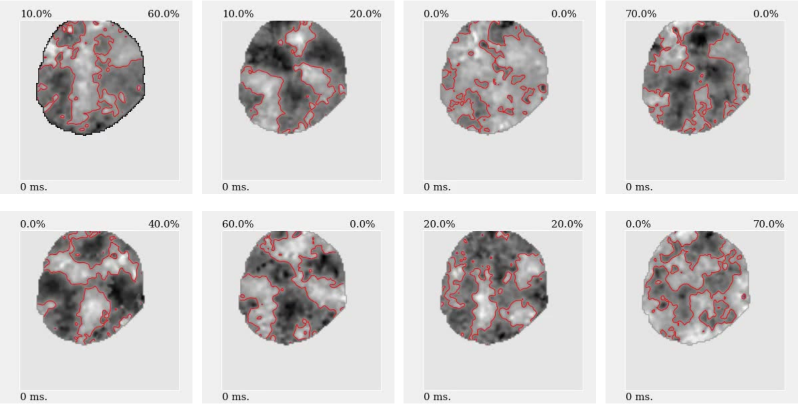


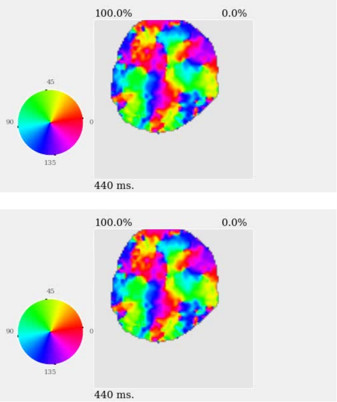


New indexes set $I_t \Rightarrow$ activation maps $m_t^{(y)}$ and orientation maps o_t for each $t \in \mathcal{T}$.

Activation maps (top: $+135^{\circ}$, bottom: $+90^{\circ}$):



Orientation maps:



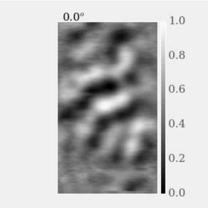
Definition

Let $\theta_0 \in \mathbb{R}/\pi\mathbb{Z}$, $\theta_1 = \theta_0 + \frac{\pi}{4}$ *and denote* $(M^{(\theta_0)}, M^{(\theta_1)})$ *the two activation maps.*

$$\forall \theta \in \mathbb{R}/\pi\mathbb{Z}, \quad Z_\theta = \left(M^{(\theta_0)} + \mathrm{i}M^{(\theta_1)}\right) \exp\left(-2\mathrm{i}(\theta - \theta_0)\right).$$

The activation map evoked by a stimulus with orientation $\theta \in \mathbb{R}/\pi\mathbb{Z}$ *is*

$$M^{(\theta)} = \mathcal{R}\mathrm{e}(Z_\theta).$$



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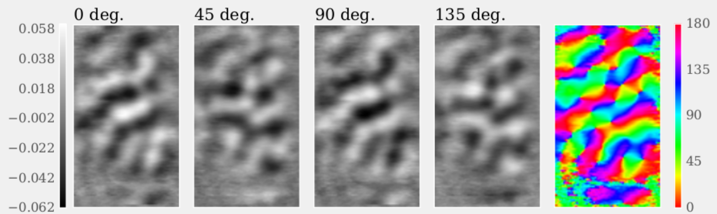

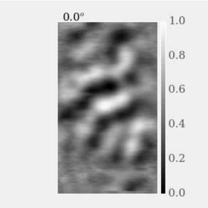

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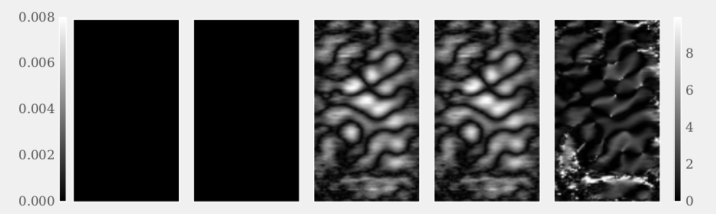

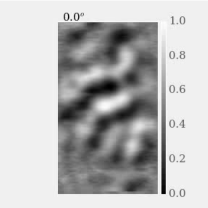

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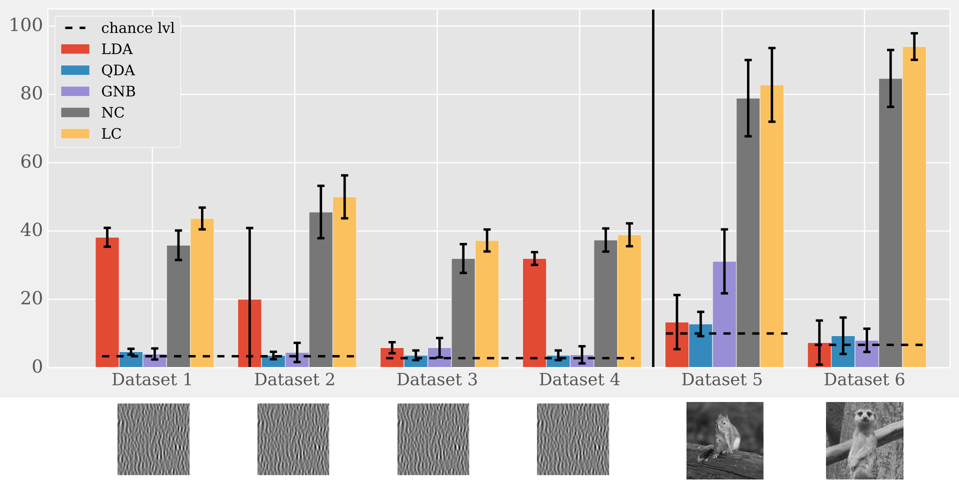
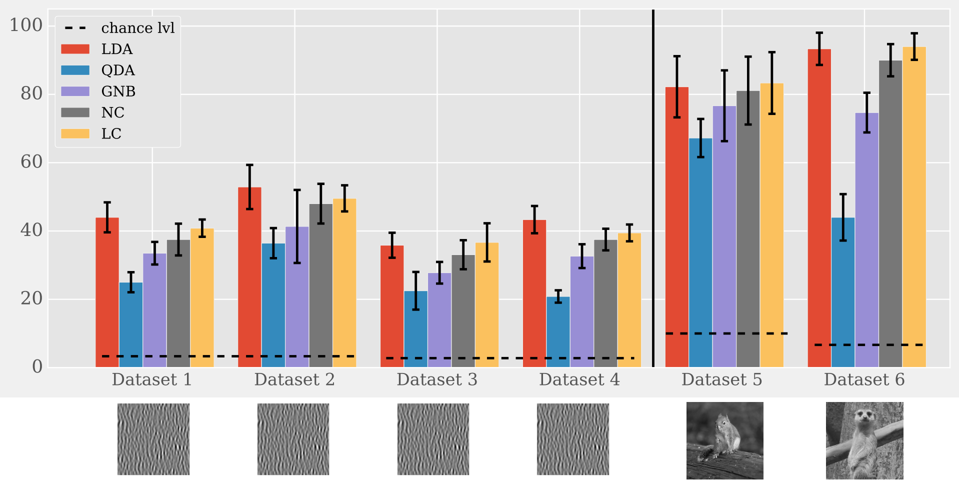
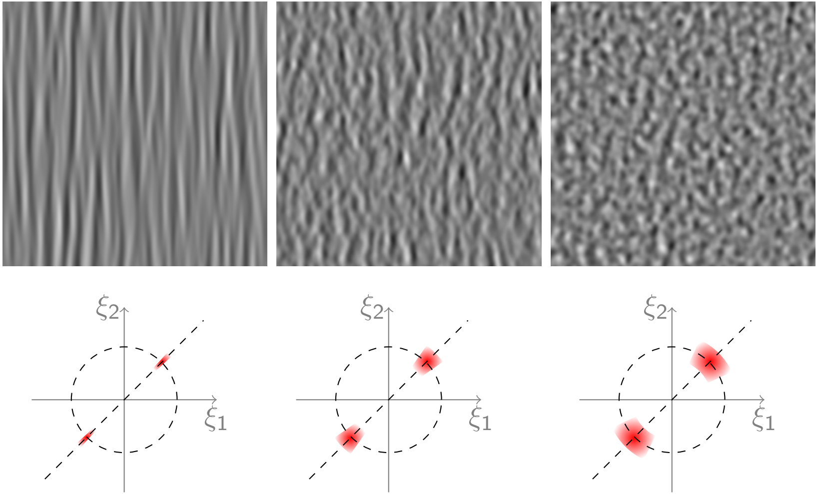


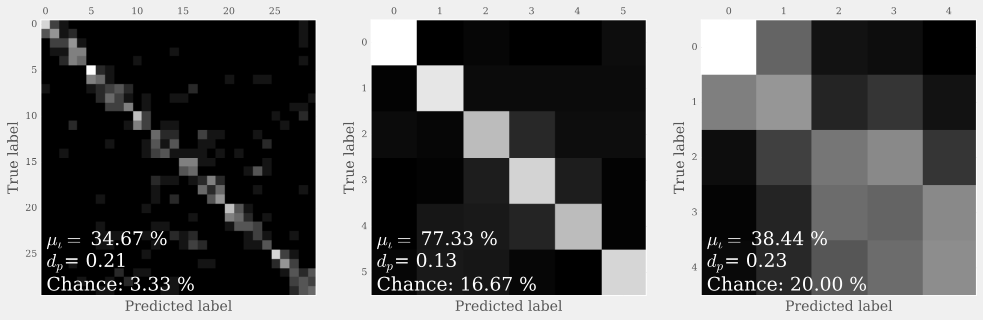
ER / Dimension Reduction and Comparison of Algorithms

- with dimension reduction LDA, QDA and GNB show better results



- 6 orientations tested
- 5 orientation bandwidths tested

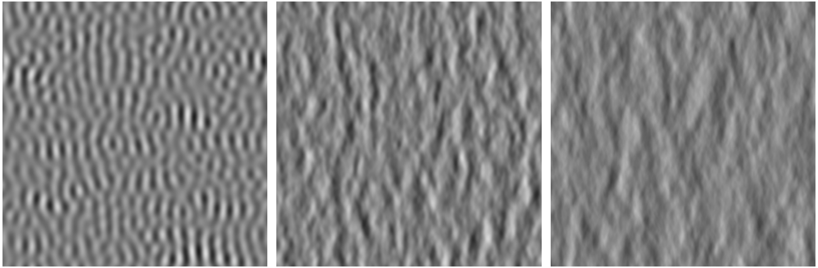


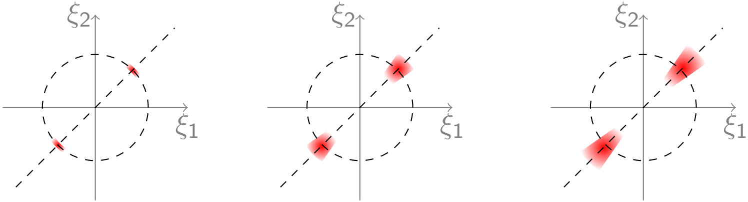


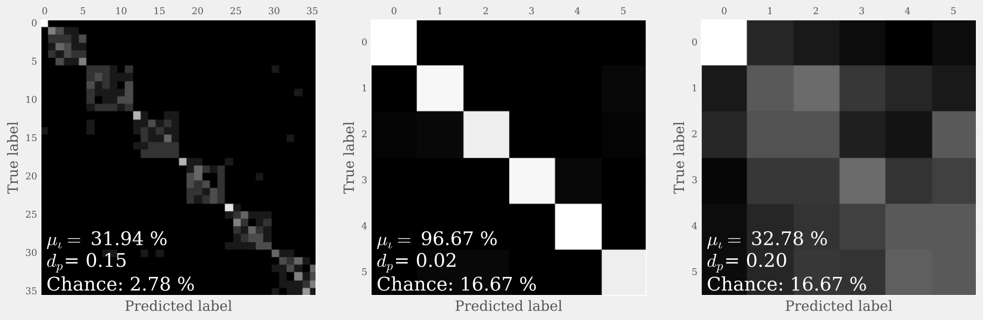
- Neural population of V1 is sensitive to orientation bandwidths

ER / Spatial Frequency Bandwidth Encoded in Neurons ?

- 6 orientations tested
- 6 spatial frequency bandwidths tested







- Neural population of V1 is sensitive to spatial frequency bandwidths

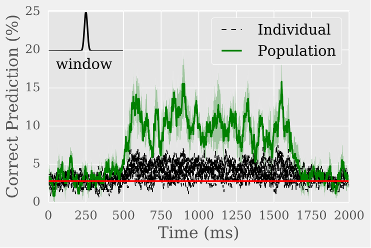
Gaussian Sliding Window

$$\forall t' \in \mathcal{T}, \quad h_t^{(1)}(t') = \exp\left(-\frac{\|t'-t\|^2}{2\sigma_h^2}\right)$$

where σ_h is the window size.

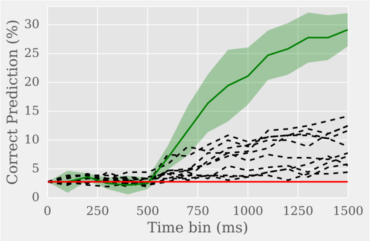
Growing Window

$$\forall t' \in \mathcal{T}, \quad h_t^{(2)}(t') = \begin{cases} 1 \text{ if } t' \leqslant t, \\ 0 \text{ else.} \end{cases}$$

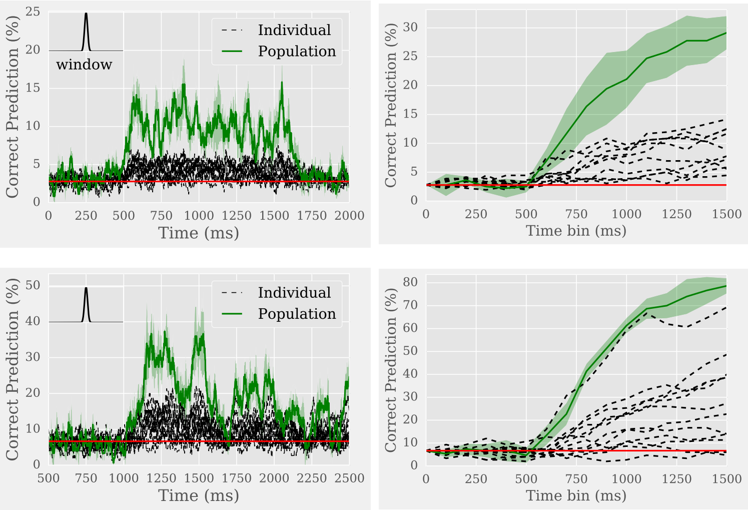


Prediction using $h_t^{(1)}\hat{\omega}_y$

Prediction using $h_t^{(2)}\hat{\omega}_y$



ER / Natural Images *vs* Motion Clouds



Motion Clouds

- Stationary predictions;
- The population improves predictions.

Natural Images

- High variability of predictions;
- A single neuron predicts as good as the entire population.

Conclusion

- **Interdisciplinary contributions** : mathematical modeling, cognitive science, experimental neurosciences;

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Conclusion

- **Interdisciplinary contributions** : mathematical modeling, cognitive science, experimental neurosciences;
- **Stochastic approaches for modeling and data analysis** : sPDE, (inverse) Bayesian inference, logistic classification;
- **Machine learning and neurosciences** : experimental protocols benefit from classification tools.

Perspectives

Dynamic textures :

- Control the parameters with respect to neural responses using Bayesian prediction models;

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Dynamic textures :

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- Make connections between Bayesian approach and LNLN models;
- Link between Bayesian priors and short time adaptation mechanisms.

The End

Thank you for your attention!

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